A Conservative Theory of Gradable Modality

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1. Introduction

Some adjectives like *important* are gradable (1a) and, as per entailments in (1b), seemingly modal in meaning:

(1) a. It is more important that Mary leave than it is that Ann stay.
b. i. $[\![\text{must}]\!]^c(p)(w) \models [\![\textit{pos}\ \text{important}]\!]^c(p)(w)$
ii. $[\![\textit{pos}\ \text{important}]\!]^c(p)(w) \models [\![\text{should}]\!]^c(p)(w)$

Theories of Gradable Modal Adjectives

		Aux WO/Q?	Adj WO/Q?
Conservative	(Portner & Rubinstein 2014)	✓	✓
Liberal	(Lassiter 2011)	✗	✗
Non-Unified	(Klecha 2014)	✓	✗

WO/Q = world ordering/quantification à la Kratzer 1981

We will restrict our investigation to conservative theories.

2. Portner & Rubinstein (2014)

Generate subsets of $g(w)$ linearly orderable by subset relation. This corresponds to the gradual removal of lower-priority premises.

Example: $g(w) = \{p, q, r\}$, subsets are $\{p, q, r\}$, $\{p, q\}$, and $\{p\}$.

Ignoring modal base for simplicity, each degree in D_a is the set of propositions true in all ideal worlds w.r.t. one of these subsets.

Example: If $\cap\{p, q, r\} \neq \emptyset$, our degrees are $d_1 = \{p' \mid p \cap q \cap r \subseteq p'\}$, $d_2 = \{p' \mid p \cap q \subseteq p'\}$, and $d_3 = \{p' \mid p \subseteq p'\}$.

Degrees are ordered by superset relation.

Example: $d_1 \supset d_2 \supset d_3$, so $d_1 <_{D_a} d_2 <_{D_a} d_3$.

$\forall$-quantifying GMAs are measure functions taking a proposition and returning the highest degree of which it is a member.

Problem 1 (Rampant Incommensurability): Let's switch the ranking of priorities p and r.

Example: $D_b = \{d_1, d_4, d_5\}$, where d_1 is as before, $d_4 = \{p' \mid q \cap r \subseteq p'\}$, $d_5 = \{p' \mid r \subseteq p'\}$, and $d_1 <_{D_b} d_4 <_{D_b} d_5$.

What if we want to compare the relative importance of q w.r.t. D_a and D_b? We need to compare d_4 to d_2, but $d_2 \not\subseteq d_4$ and $d_4 \not\subseteq d_2$. More generally, we predict near-universal incommensurability when comparing across ordering sources/prioritizations. But examples like (2) are fine:

(2) *(Context: Bill's company changed its dress code on Tuesday.)*
(In view of his company's dress code,) It is as important that Bill wear his suit to work today as it was on Monday.

Problem 2 (Incompatibles): As P&R note, this theory can't account for equally important incompatible propositions like in (3):

(3) a. It is as important to preserve the wetlands as it is to build the new housing (which would drain the wetlands). (P&R 2014)

P&R suggest we abandon the view that *important* involves $\forall$-quantification. But this doesn't solve Problem 1, and we lose an insight: **Problem 1 = Problem 2.**

(3) b. It is as important *(in view of our environmental priorities)* to preserve the wetlands as it is *(in view of our financial priorities)* to build the new housing.

3. A New Conservative Theory: Overview

- Include modal degrees in the ontology, with ordering $\leq_D$.
- Ordering source is a function from worlds **and degrees** to sets of propositions. In other words, the ordering source is **degree-relative**.
- World-orderings from higher degrees "trickle down" to lower degrees, similar to von Fintel & Iatridou's (2008) treatment of weak necessity modals like *should*.
- By keeping our degrees fixed and adjusting the ordering source around them, we can allow for comparisons across ordering sources.

4. World-Ordering

At a single degree, world-ordering works just like Kratzer 1981:

(4) $w_1 \leq_{g,w,d} w_2$ iff $\{p \in g(w)(d) \mid p(w_1)\} \supseteq \{p \in g(w)(d) \mid p(w_2)\}$

$\leq^+_{g,w,d}$ orders worlds w.r.t. all degrees $\geq_D d$ (via trickle-down):

(5) $w_1 \leq^+_{g,w,d} w_2$ iff $w_1 <^+_{g,w,d} w_2$ or $w_1 =^+_{g,w,d} w_2$, where:

a. $w_1 <^+_{g,w,d} w_2$ iff $\exists d' \geq_D d[w_1 <_{g,w,d'} w_2 \wedge \forall d'' \geq_D d'[w_1 \leq_{g,w,d''} w_2]]$

$w_1 \leq_{g,w,d_4} w_2$	d_4	
$w_1 \leq_{g,w,d_3} w_2$	d_3	
$w_1 <_{g,w,d_2} w_2$	d_2	$w_1 <^+_{g,w,d_2} w_2$
doesn't matter	d_1	$w_1 <^+_{g,w,d_1} w_2$

b. $w_1 =^+_{g,w,d} w_2$ iff $\forall d' \geq_D d[w_1 =_{g,w,d'} w_2]$

$w_1 =_{g,w,d_4} w_2$	d_4	$w_1 =^+_{g,w,d_4} w_1$
$w_1 =_{g,w,d_3} w_2$	d_3	$w_1 =^+_{g,w,d_3} w_1$
$w_1 \neq_{g,w,d_2} w_2$	d_2	$w_1 \neq^+_{g,w,d_2} w_2$
doesn't matter	d_1	$w_1 \neq^+_{g,w,d_1} w_2$

End result: Strict ordering and incomparability trickle down to lower degrees, while equivalence does not.

5. A First Crack at *Important*

Our definition of $\leq^+$ entails the following:

If $\text{Best}(g, f, w, d) = \{w' \in \cap f(w) \mid \neg\exists w'' \in \cap f(w)[w'' <^+_{g,w,d} w']\}$, then if $d_1 <_D d_2$, $\text{Best}(g, f, w, d_1) \subseteq \text{Best}(g, f, w, d_2)$. Hence, as we go down the scale, $\forall$-quantification gets weaker.

Treating the denotation of *important* as a measure function:

(6) $[\![\text{important}]\!]^c = \lambda p \lambda w.\ \sup(\{d \mid \forall w' \in \text{Best}(g^c, f^c, w, d)[p(w')]\})$, where $\sup(\delta)$ is the supremum (least upper bound) of δ

If g_{env} = environmental priorities and g_{fin} = financial priorities, then:

(7) $[\![(3)]\!] = \lambda w.\sup(\{d \mid \forall w' \in \text{Best}(g_{env}, f, w, d)[\text{wetlands preserved in } w']\}) \geq \sup(\{d \mid \forall w' \in \text{Best}(g_{fin}, f, w, d)[\text{housing built in } w']\})$

6. *Permissible* and Cross-Polarity

Note that (8a) and (8b) are mutually entailing:

(8) a. It is more important that p than it is that q.
b. It is more permissible that not-q than it is that not-p.

This looks similar to cross-polarity: *important* and *permissible* are like *tall*/*short*, except for the negation in (8b). We can capture this by means of the dual nature of $\forall$- and $\exists$-quantification.

I follow Kennedy (1997) in switching from single degrees to *extents* (closed intervals of degrees). I treat the denotations of *important* and *permissible* as upward and downward extents (respectively), exploiting the following fact:

Due to the definition of $\leq^+$, while $\forall$-quantification gets stronger as we go **up** the scale, $\exists$-quantification gets stronger as we go **down**.

If $\text{close}(\delta)$ is the closure of δ (i.e., the smallest closed superset of δ), we can (re)define *important* and *permissible* as follows:

(9) a. Definition of *important* (revised) and *permissible*:
i. $[\![\text{important}]\!]^c = \lambda p \lambda w.\ \text{close}(\{d \mid \forall w' \in \text{Best}(g^c, f^c, w, d)[p(w')]\})$
ii. $[\![\text{permissible}]\!]^c = \lambda p \lambda w.\ \text{close}(\{d \mid \exists w' \in \text{Best}(g^c, f^c, w, d)[p(w')]\})$
b. If $[\![\text{important}]\!]^c(p)(w) \neq \emptyset$ and $[\![\text{permissible}]\!]^c(p)(w) \neq \emptyset$, these are equivalent to:
i. $\{d \mid d \leq_D \sup(\{d' \mid \forall w' \in \text{Best}(g^c, f^c, w, d')[p(w')]\})\}$
ii. $\{d \mid d \geq_D \inf(\{d' \mid \exists w' \in \text{Best}(g^c, f^c, w, d')[p(w')]\})\}$, where $\inf(\delta)$ is the infimum (greatest lower bound) of δ.

As a result, $[\![\text{important}]\!]^c(p)(w)$ gives us a (possibly empty) interval from the bottom upward, while $[\![\text{permissible}]\!]^c(p)(w)$ gives us a (possibly empty) interval from the top downward.

Assuming $[\![\text{-er}]\!]$ is the proper superset relation, this gets us the entailments in (8), as (10a-b) are equivalent:

(10) a. $[\![(8a)]\!]^c = \lambda w.\text{close}(\{d \mid \forall w' \in \text{Best}(g^c, f^c, w, d)[p(w')]\}) \supset \text{close}(\{d \mid \forall w' \in \text{Best}(g^c, f^c, w, d)[q(w')]\})$
b. $[\![(8b)]\!]^c = \lambda w.\text{close}(\{d \mid \exists w' \in \text{Best}(g^c, f^c, w, d)[\neg q(w')]\}) \supset \text{close}(\{d \mid \exists w' \in \text{Best}(g^c, f^c, w, d)[\neg p(w')]\})$

Plus, if *important* and *permissible* are cross-polar, we rightly predict (11) to be ill-formed due to cross-polar anomaly (cf. Kennedy 1997):

(11) # It is more permissible that Ann stay than it is important that Mary go.

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