

1. Introduction

Some adjectives like *important* are gradable (1a) and, as per entailments in (1b), seemingly modal in meaning:

- (1) a. It is more important that Mary leave than it is that Ann stay.
 b. i. $\llbracket \text{must} \rrbracket^c(p)(w) \models \llbracket \text{pos important} \rrbracket^c(p)(w)$
 ii. $\llbracket \text{pos important} \rrbracket^c(p)(w) \models \llbracket \text{should} \rrbracket^c(p)(w)$

Theories of Gradable Modal Adjectives

		Aux WO/Q?	Adj WO/Q?
Conservative	(Portner & Rubinstein 2014)	✓	✓
Liberal	(Lassiter 2011)	✗	✗
Non-Unified	(Klecha 2014)	✓	✗

WO/Q = world ordering/quantification à la Kratzer 1981

We will restrict our investigation to conservative theories.

2. Portner & Rubinstein (2014)

Generate subsets of $g(w)$ linearly orderable by subset relation. This corresponds to the gradual removal of lower-priority premises.

Example: $g(w) = \{p, q, r\}$, subsets are $\{p, q, r\}$, $\{p, q\}$, and $\{p\}$.

Ignoring modal base for simplicity, each degree in D_a is the set of propositions true in all ideal worlds w.r.t. one of these subsets.

Example: If $\cap\{p, q, r\} \neq \emptyset$, our degrees are $d_1 = \{p' \mid p \cap q \cap r \subseteq p'\}$, $d_2 = \{p' \mid p \cap q \subseteq p'\}$, and $d_3 = \{p' \mid p \subseteq p'\}$.

Degrees are ordered by superset relation.

Example: $d_1 \supset d_2 \supset d_3$, so $d_1 <_{D_a} d_2 <_{D_a} d_3$.

\forall -quantifying GMAs are measure functions taking a proposition and returning the highest degree of which it is a member.

PROBLEM 1 (RAMPANT INCOMMENSURABILITY): Let's switch the ranking of priorities p and r .

Example: $D_b = \{d_1, d_4, d_5\}$, where d_1 is as before, $d_4 = \{p' \mid q \cap r \subseteq p'\}$, $d_5 = \{p' \mid r \subseteq p'\}$, and $d_1 <_{D_b} d_4 <_{D_b} d_5$.

What if we want to compare the relative importance of q w.r.t. D_a and D_b ? We need to compare d_4 to d_2 , but $d_2 \not\subseteq d_4$ and $d_4 \not\subseteq d_2$. More generally, we predict near-universal incommensurability when comparing across ordering sources/prioritizations. But examples like (2) are fine:

- (2) (Context: Bill's company changed its dress code on Tuesday.)
 (In view of his company's dress code,) It is as important that Bill wear his suit to work today as it was on Monday.

PROBLEM 2 (INCOMPATIBLES): As P&R note, this theory can't account for equally important incompatible propositions like in (3):

- (3) a. It is as important to preserve the wetlands as it is to build the new housing (which would drain the wetlands). (P&R 2014)

P&R suggest we abandon the view that *important* involves \forall -quantification. But this doesn't solve Problem 1, and we lose an insight:

Problem 1 = Problem 2.

- (3) b. It is as important (in view of our environmental priorities) to preserve the wetlands as it is (in view of our financial priorities) to build the new housing.

3. A New Conservative Theory: Overview

- Include modal degrees in the ontology, with ordering \leq_D .
- Ordering source is a function from worlds **and** degrees to sets of propositions. In other words, the ordering source is **degree-relative**.
- World-orderings from higher degrees "trickle down" to lower degrees, similar to von Stechow & Iatridou's (2008) treatment of weak necessity modals like *should*.
- By keeping our degrees fixed and adjusting the ordering source around them, we can allow for comparisons across ordering sources.

4. World-Ordering

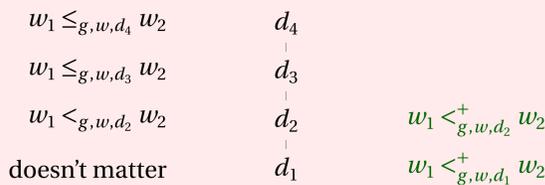
At a single degree, world-ordering works just like Kratzer 1981:

- (4) $w_1 \leq_{g,w,d} w_2$ iff $\{p \in g(w)(d) \mid p(w_1)\} \supseteq \{p \in g(w)(d) \mid p(w_2)\}$

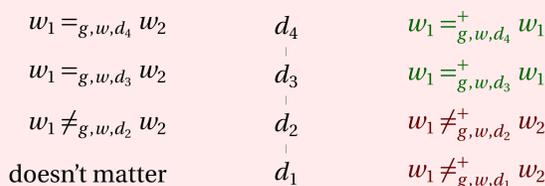
$\leq_{g,w,d}^+$ orders worlds w.r.t. all degrees $\geq_D d$ (via trickle-down):

- (5) $w_1 \leq_{g,w,d}^+ w_2$ iff $w_1 <_{g,w,d}^+ w_2$ or $w_1 =_{g,w,d}^+ w_2$, where:

- a. $w_1 <_{g,w,d}^+ w_2$ iff $\exists d' \geq_D d [w_1 <_{g,w,d'} w_2 \wedge \forall d'' \geq_D d' [w_1 \leq_{g,w,d''} w_2]]$



- b. $w_1 =_{g,w,d}^+ w_2$ iff $\forall d' \geq_D d [w_1 =_{g,w,d'} w_2]$



End result: Strict ordering and incomparability trickle down to lower degrees, while equivalence does not.

5. A First Crack at *Important*

Our definition of \leq^+ entails the following:

If $\text{BEST}(g, f, w, d) = \{w' \in \cap f(w) \mid \neg \exists w'' \in \cap f(w) [w'' <_{g,w,d}^+ w']\}$, then if $d_1 <_D d_2$, $\text{BEST}(g, f, w, d_1) \subseteq \text{BEST}(g, f, w, d_2)$. Hence, as we go down the scale, \forall -quantification gets weaker.

Treating the denotation of *important* as a measure function:

- (6) $\llbracket \text{important} \rrbracket^c = \lambda p \lambda w. \sup(\{d \mid \forall w' \in \text{BEST}(g^c, f^c, w, d)[p(w')]\})$, where $\sup(\delta)$ is the supremum (least upper bound) of δ

If g_{env} = environmental priorities and g_{fin} = financial priorities, then:

- (7) $\llbracket (3) \rrbracket = \lambda w. \sup(\{d \mid \forall w' \in \text{BEST}(g_{env}, f, w, d)[\text{wetlands preserved in } w']\} \supseteq \sup(\{d \mid \forall w' \in \text{BEST}(g_{fin}, f, w, d)[\text{housing built in } w']\})$

6. Permissible and Cross-Polarity

Note that (8a) and (8b) are mutually entailing:

- (8) a. It is more important that p than it is that q .
 b. It is more permissible that not- q than it is that not- p .

This looks similar to cross-polarity: *important* and *permissible* are like *tall/short*, except for the negation in (8b). We can capture this by means of the dual nature of \forall - and \exists -quantification.

I follow Kennedy (1997) in switching from single degrees to *extents* (closed intervals of degrees). I treat the denotations of *important* and *permissible* as upward and downward extents (respectively), exploiting the following fact:

Due to the definition of \leq^+ , while \forall -quantification gets stronger as we go **up** the scale, \exists -quantification gets stronger as we go **down**.

If $\text{close}(\delta)$ is the closure of δ (i.e., the smallest closed superset of δ), we can (re)define *important* and *permissible* as follows:

- (9) a. Definition of *important* (revised) and *permissible*:
 i. $\llbracket \text{important} \rrbracket^c = \lambda p \lambda w. \text{close}(\{d \mid \forall w' \in \text{BEST}(g^c, f^c, w, d)[p(w')]\})$
 ii. $\llbracket \text{permissible} \rrbracket^c = \lambda p \lambda w. \text{close}(\{d \mid \exists w' \in \text{BEST}(g^c, f^c, w, d)[p(w')]\})$
 b. If $\llbracket \text{important} \rrbracket^c(p)(w) \neq \emptyset$ and $\llbracket \text{permissible} \rrbracket^c(p)(w) \neq \emptyset$, these are equivalent to:
 i. $\{d \mid d \leq_D \sup(\{d' \mid \forall w' \in \text{BEST}(g^c, f^c, w, d')[p(w')]\})\}$
 ii. $\{d \mid d \geq_D \inf(\{d' \mid \exists w' \in \text{BEST}(g^c, f^c, w, d')[p(w')]\})\}$, where $\inf(\delta)$ is the infimum (greatest lower bound) of δ .

As a result, $\llbracket \text{important} \rrbracket^c(p)(w)$ gives us a (possibly empty) interval from the bottom upward, while $\llbracket \text{permissible} \rrbracket^c(p)(w)$ gives us a (possibly empty) interval from the top downward.

Assuming $\llbracket \text{-er} \rrbracket$ is the proper superset relation, this gets us the entailments in (8), as (10a-b) are equivalent:

- (10) a. $\llbracket (8a) \rrbracket^c = \lambda w. \text{close}(\{d \mid \forall w' \in \text{BEST}(g^c, f^c, w, d)[p(w')]\}) \supseteq \text{close}(\{d \mid \forall w' \in \text{BEST}(g^c, f^c, w, d)[q(w')]\})$
 b. $\llbracket (8b) \rrbracket^c = \lambda w. \text{close}(\{d \mid \exists w' \in \text{BEST}(g^c, f^c, w, d)[\neg q(w')]\}) \supseteq \text{close}(\{d \mid \exists w' \in \text{BEST}(g^c, f^c, w, d)[\neg p(w')]\})$

Plus, if *important* and *permissible* are cross-polar, we rightly predict (11) to be ill-formed due to cross-polar anomaly (cf. Kennedy 1997):

- (11) # It is more permissible that Ann stay than it is important that Mary go.

Acknowledgments

Many thanks to Paola C epeda, Thomas Graf, Richard Larson, So Young Lee, Lei Liu, Yaobin Liu, Paul Portner, Aynat Rubinstein, Ildik o Emese Szab o, Chong Zhang, and audiences at Stony Brook University, NYU, and the Workshop on Modality Across Categories (Pompeu Fabra University).

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